ROM2F-96/34
hep-th/9606169
February 1, 2008

Chiral Asymmetry in Four-Dimensional Open-String Vacua

C. Angelantonj

Dipartimento di Fisica
Università dell' Aquila
I.N.F.N. - L.N.G.S.
67010 Coppito (AQ) ITALY

M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev[1]

Dipartimento di Fisica
Università di Roma "Tor Vergata"
I.N.F.N. - Sezione di Roma "Tor Vergata"
Via della Ricerca Scientifica , 1
00133 Roma ITALY

Abstract

Starting from the type IIB string on the Z orbifold, we construct some chiral open-string vacua with $N = 1$ supersymmetry in four dimensions. The Chan-Paton group depends on the (quantized) NS-NS antisymmetric tensor. The largest choice, $SO(8) \times SU(12) \times U(1)$, has an anomalous $U(1)$ factor whose gauge boson acquires a mass of the order of the string scale. The corresponding open-string spectrum comprises only Neumann strings and includes three families of chiral multiplets in the $(\mathbf{8}, \mathbf{12}^*)+(\mathbf{1}, \mathbf{66})$ representation. A comparison is drawn with a heterotic vacuum with non-standard embedding, and some properties of the low-energy effective field theory are discussed.

[1]I.N.F.N. Fellow, on Leave from Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, BG-1784 Sofia, BULGARIA.

1 Introduction

Calabi-Yau (CY) compactifications of the type IIB string yield $N = 2$ supergravity in four dimensions with $h^{(1,1)} + 1$ hypermultiplets and $h^{(1,2)}$ vector multiplets, and are naturally related via a world-sheet orbifold procedure [1] to four-dimensional open-string vacua with $N = 1$ supersymmetry. At rational points of their moduli spaces, many CY compactifications have microscopic descriptions as Gepner models, tensor products of $N = 2$ superconformal models [2]. Both these models and $N = 1$ superconformal models of free fermions [3] are often related to orbifolds of tori [4] by continuous deformations. Open descendants of rational models have been studied rather extensively in six dimensions [5, 6, 7], and have displayed a variety of new interesting phenomena [8]. The simple rational setting of [5], however, is not convenient to explore the four-dimensional models, since typical rational lattices involve (quantized) background values of the NS-NS antisymmetric tensor that reduce [9] the size of the Chan-Paton (CP) group. Therefore, in this letter we return to the original setting of [10], essentially equivalent to [6], and construct chiral four-dimensional models related to the type IIB string on the Z orbifold. For notational simplicity, in all amplitudes we omit measure factors and contributions inert under the GSO projections.

2 The Z Orbifold and the Type I Superstring

In order to construct the open descendants of the type IIB superstring on the Z orbifold, let us briefly recall the building blocks of the original construction of [4]. One starts from the compactification on a six-torus consisting of three orthogonal copies of a two-dimensional hexagonal lattice with scalar product

$$(e_a, e_b) = G_{ab} = \frac{R^2}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad . \tag{2.1}$$

The corresponding type IIB partition function is

$$T = |V_8 - S_8|^2 \left(\frac{\sum\, q^{\frac{\alpha'}{4} p_{La} G^{ab} p_{Lb}}\ \bar{q}^{\frac{\alpha'}{4} p_{Ra} G^{ab} p_{Rb}}}{\eta^2(q)\eta^2(\bar{q})} \right)^3 \quad , \tag{2.2}$$

where V_8 and S_8 are level-one $SO(8)$ characters, $(p_a)_{L,R} = m_a \pm \frac{1}{2} G_{ab} n^b$ and $q = e^{2\pi i \tau}$. As anticipated, we begin with a vanishing NS-NS tensor B in order to obtain a CP group of maximum size.

The Z orbifold construction uses the natural Z_3 action on the three complex internal coordinates $X^i \sim \omega X^i$, where $\omega = e^{\frac{2i\pi}{3}}$, that results in a total of 27 fixed points. In particular, the projection of the untwisted sector may be expressed in terms of

$$H_{0,\epsilon}(q) = q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 - \omega^\epsilon q^n)(1 - \bar{\omega}^\epsilon q^n) \quad , \tag{2.3}$$

with $\epsilon = 0, \pm 1$, while the twisted sectors may be expressed in terms of

$$H_{+,\epsilon}(q) = H_{-,-\epsilon}(q) = 3^{-\frac{1}{2}} q^{-\frac{1}{36}} \prod_{n=0}^{\infty} (1 - \omega^\epsilon q^{n+\frac{1}{3}})(1 - \bar{\omega}^\epsilon q^{n+\frac{2}{3}}) \quad . \tag{2.4}$$

Maintaining $N = 2$ space-time supersymmetry in the type IIB superstring requires that twists on internal bosonic coordinates, X^i, be properly correlated with twists on internal world-sheet fermions, ψ^i [4]. The level-one $SO(8)$ characters V_8 and S_8 associated to the transverse world-sheet fermions are thus to be decomposed with respect to $SO(2) \times SU(3) \times U(1)$. To this end, it is convenient to introduce the level-one $SU(3)$ characters $\{\chi_0, \chi_+, \chi_-\}$, of conformal weights $\{0, 1/3, 1/3\}$, and to define the supersymmetric characters $\{A_0, A_+, A_-\}$, of conformal weights $\{1/2, 1/6, 1/6\}$. The ten-dimensional GSO projection then translates into

$$V_8 - S_8 = A_0 \chi_0 + A_+ \chi_- + A_- \chi_+ \quad . \tag{2.5}$$

A_0, A_+ and A_- may be expressed in terms of the four level-one $SO(2)$ characters and of the 12 characters ξ_m ($m = -5, \ldots, 6$) of the $N = 2$ superconformal model with $c = 1$ (equivalent to the rational torus at radius $R = \sqrt{12}$), of conformal weight $h_m = \frac{m^2}{24}$, as

$$\begin{aligned} A_0 &= V_2 \xi_0 + O_2 \xi_6 - S_2 \xi_{-3} - C_2 \xi_3 \\ A_+ &= V_2 \xi_4 + O_2 \xi_{-2} - S_2 \xi_1 - C_2 \xi_{-5} \\ A_- &= V_2 \xi_{-4} + O_2 \xi_2 - S_2 \xi_5 - C_2 \xi_{-1} \quad . \end{aligned} \tag{2.6}$$

The spectrum of the "parent" type IIB string on the Z orbifold can thus be extracted

from the torus amplitude

$$
\begin{aligned}
T &= \frac{1}{3}\,\Xi_{0,0}(q)\;\Xi_{0,0}(\bar{q})\sum q^{\frac{\alpha'}{4}p_{La}G^{ab}p_{Lb}}\bar{q}^{\frac{\alpha'}{4}p_{Ra}G^{ab}p_{Rb}} + \frac{1}{3}\sum_{\epsilon=\pm 1}\Xi_{0,\epsilon}(q)\;\Xi_{0,\epsilon}(\bar{q}) \\
&\quad + \frac{1}{3}\sum_{\eta=\pm 1}\;\sum_{\epsilon=0,\pm 1}\Xi_{\eta,\epsilon}(q)\;\Xi_{-\eta,-\epsilon}(\bar{q}) \quad , \qquad (2.7)
\end{aligned}
$$

where

$$
\begin{aligned}
\Xi_{0,\epsilon}(q) &= \left(\frac{A_0\chi_0 + \omega^{\epsilon}A_+\chi_- + \bar{\omega}^{\epsilon}A_-\chi_+}{{H_{0,\epsilon}}^3}\right)(q) \\
\Xi_{+,\epsilon}(q) &= \left(\frac{A_0\chi_+ + \omega^{\epsilon}A_+\chi_0 + \bar{\omega}^{\epsilon}A_-\chi_-}{{H_{+,\epsilon}}^3}\right)(q) \\
\Xi_{-,\epsilon}(q) &= \left(\frac{A_0\chi_- + \omega^{\epsilon}A_-\chi_0 + \bar{\omega}^{\epsilon}A_+\chi_+}{{H_{-,\epsilon}}^3}\right)(q) \quad . \qquad (2.8)
\end{aligned}
$$

The massless untwisted states comprise the $N = 2$ supergravity multiplet, the universal dilaton hypermultiplet and 9 additional hypermultiplets. The twisted sectors account for 27 hypermultiplets, one for each fixed point. Thus, the massless spectrum corresponds to a CY compactification with Hodge numbers $h^{(1,1)} = 36$ and $h^{(1,2)} = 0$. Two of the four scalars in each hypermultiplet, however, arise from the R-R sector. Since their emission vertices vanish at zero momentum, it seems more appropriate to describe them in terms of complex antisymmetric tensors $B^{(I)}_{\mu\nu}$, with $I = 0, 1, .., h^{(1,1)}$ [11].

In constructing the open descendants, one starts by halving the torus amplitude of eq. (2.7). Since the Z_3 action of the target space twist is L-R symmetric, the Klein-bottle amplitude is

$$
K = \frac{1}{6}\,\Xi_{0,0}(q^2)\;\sum\;q^{\frac{\alpha'}{2}m_aG^{ab}m_b} \;+\; \frac{1}{6}\,\Xi_{0,+}(q^2) \;+\; \frac{1}{6}\,\Xi_{0,-}(q^2) \quad , \qquad (2.9)
$$

where $q = \exp(-2\pi\tau_2)$, with τ_2 the closed-string "proper time". Eq. (2.9) contains only the conventional sum over the momentum lattice since, for generic values of R, the condition ${p_L}^{\omega} = p_R$ does not have any non-trivial solutions. One may thus anticipate that the open sector includes only Neumann charges associated with the ubiquitous nine-branes. This should be contrasted with the Z_2 case [10, 6], where additional contributions to K signal the appearance of D charges in the open spectrum. The massless states in the projected closed-string spectrum thus comprise the $N = 1$ supergravity multiplet, a

universal linear multiplet and 9 additional linear multiplets from the untwisted sector, as well as 27 linear multiplets from the twisted sectors.

The twisted sector of the world-sheet orbifold, to be identified with the open-string spectrum, starts with the annulus amplitude

$$
\begin{aligned}
A &= \frac{(\mathcal{N}+\mathcal{M}+\bar{\mathcal{M}})^2}{6}\, \Xi_{0,0}(\sqrt{q}) \sum q^{\frac{\alpha'}{2} m_a G^{ab} m_b} \\
&\quad + \frac{(\mathcal{N}+\omega\mathcal{M}+\bar{\omega}\bar{\mathcal{M}})^2}{6}\, \Xi_{0,+}(\sqrt{q}) + \frac{(\mathcal{N}+\bar{\omega}\mathcal{M}+\omega\bar{\mathcal{M}})^2}{6}\, \Xi_{0,-}(\sqrt{q}) \quad , \qquad (2.10)
\end{aligned}
$$

where $\mathcal{N}, \mathcal{M}, \bar{\mathcal{M}}$ are CP multiplicities. The Möbius amplitude presents some subtleties connected with the proper definition of a set of real "hatted" characters [5]. Since both the twisted sectors and the projections of the untwisted sector are independent of the moduli R_i, it proves convenient to exploit the enhanced $SU(3)^3$ symmetry at the rational point $R_i = \sqrt{3}$ to define

$$
\begin{aligned}
\hat{\Xi}_{0,\epsilon} &= \left(\hat{A}_0\hat{\chi}_0 + \omega^\epsilon \hat{A}_+\hat{\chi}_- + \bar{\omega}^\epsilon \hat{A}_-\hat{\chi}_+\right)\left(\hat{\chi}_0^3 - \omega\hat{\chi}_\epsilon^3 - \bar{\omega}\hat{\chi}_{-\epsilon}^3\right) \\
\hat{\Xi}_{\epsilon,0} &= \left(\hat{A}_0\hat{\chi}_\epsilon + \hat{A}_\epsilon\hat{\chi}_0 - \hat{A}_{-\epsilon}\hat{\chi}_{-\epsilon}\right)\left(\hat{\chi}_{-\epsilon}\hat{\chi}_0^2 + \hat{\chi}_\epsilon\hat{\chi}_{-\epsilon}^2 - \hat{\chi}_0\hat{\chi}_\epsilon^2\right) \quad , \qquad (2.11)
\end{aligned}
$$

with $\epsilon = \pm 1$. This choice of signs defines a flip operator for open strings that ensures the compatibility of direct and transverse Möbius channels, related by the transformation $P = T^{1/2} S T^2 S T^{1/2}$, that simply interchanges $\hat{\Xi}_{0,\epsilon}$ and $\hat{\Xi}_{-\epsilon,0}$. One may then verify that

$$
\begin{aligned}
M &= -\frac{(\mathcal{N}+\mathcal{M}+\bar{\mathcal{M}})}{6}\, \hat{\Xi}_{0,0}(-\sqrt{q}) \sum q^{\frac{\alpha'}{2} m_a G^{ab} m_b} \\
&\quad - \frac{(\mathcal{N}+\bar{\omega}\mathcal{M}+\omega\bar{\mathcal{M}})}{6}\, \hat{\Xi}_{0,+}(-\sqrt{q}) - \frac{(\mathcal{N}+\omega\mathcal{M}+\bar{\omega}\bar{\mathcal{M}})}{6}\, \hat{\Xi}_{0,-}(-\sqrt{q}) \qquad (2.12)
\end{aligned}
$$

completes the open sector of the spectrum, while the three vacuum-channel amplitudes $\tilde{K}$, $\tilde{A}$ and $\tilde{M}$ are compatible with factorization. Demanding that unphysical massless states decouple from the transverse channel leads to the two tadpole conditions

$$
\begin{aligned}
\mathcal{N}+\mathcal{M}+\bar{\mathcal{M}} &= 32 \\
\mathcal{N} - \frac{1}{2}(\mathcal{M}+\bar{\mathcal{M}}) &= -4 \quad , \qquad (2.13)
\end{aligned}
$$

related to untwisted and twisted massless exchanges respectively. Together with (2.12), eqs. (2.13) yield a CP gauge group $SO(8) \times SU(12) \times U(1)$. Eqs. (2.10) and (2.12) and

the definitions in (2.8) then imply that the open sector contains three generations of chiral multiplets in the representations $(\mathbf{8}, \mathbf{12}^*_{-\mathbf{1}})$ and $(\mathbf{1}, \mathbf{66}_{\mathbf{2}})$. Tadpole cancellation guarantees that this chiral spectrum is anomaly free, aside from the $U(1)$ factor.

3 The Anomalous $U(1)$

As pointed out in [12], the $U(1)$ anomaly translates into a Higgs-like mechanism that gives the $U(1)$ gauge field a mass of the order of the string scale M_s. Denoting by tr the trace in the fundamental representation, the anomaly six-form is

$$A_6 \sim F\left(\frac{1}{12}F^2 + tr(F_{12}^2) - tr(F_8^2)\right) \quad , \tag{3.1}$$

where F, F_{12} and F_8 are field strengths of $U(1)$, $SU(12)$ and $SO(8)$. The resulting consistent anomaly is canceled by a Green-Schwarz mechanism induced by $L_{GS} = B \wedge F$ and by the Chern-Simons coupling

$$H = dB + \frac{1}{12}A \wedge F + \omega_{3(12)} - \omega_{3(8)} \quad . \tag{3.2}$$

It is convenient to perform a duality transformation, turning $B_{\mu\nu}$ into a pseudo-scalar axion β, and combine it with the dilaton ϕ into a complex scalar $S = \beta + ie^{-2\phi}$. In the heterotic string, at tree level S describes a non-linear σ-model on the coset $SL(2,R)/U(1)$ with Kähler potential $K = -\log(S - \bar{S})$ and enters the gauge kinetic function $f_{ab} = S\delta_{ab}$ [14]. Gauging the holomorphic isometry generated by $\partial/\partial\beta$ leads to the coupling $\partial_\mu\beta \rightarrow D_\mu\beta = \partial_\mu\beta + kA_\mu$, and the gauge variation of β, $\delta\beta = -k\alpha$, qualifies it as a Stückelberg field. The supersymmetric completion of these couplings is simply $K = -\log(S - \bar{S} + ikV)$.

In compactifications of the $SO(32)$ heterotic string with standard embedding, the (generically) anomalous spectrum of $U(1)$ charges is responsible for the generation of a Fayet-Iliopoulos D-term at one-loop. Massless scalars charged under $U(1)$ that are singlets of the unbroken gauge group $SO(26) \times SU(3)$ allow one to shift the vacuum with broken supersymmetry to a nearby supersymmetric vacuum, giving masses to $U(1)$ charged particles as well to the $U(1)$ gauge field [12]. Other would-be axions cannot acquire couplings similar to β because of non-renormalization theorems [12].

Although it is reasonable to expect that anomaly related terms in the effective lagrangian turn into one another under heterotic - type I duality [15], the simple strong - weak coupling duality $\phi_I = -\phi_H$ in $d = 10$ [17] (that would already affect the loop-counting arguments of [12]) changes deeply in lower dimensions. Indeed, since the dilaton belongs to the universal sector of the compactification, the relation between the heterotic and type I dilaton in d dimensions is determined by dimensional reduction to be

$$\phi_I^{(d)} \;=\; \frac{6-d}{4}\,\phi_H^{(d)} \;-\; \frac{(d-2)}{16}\log\det G_H^{(10-d)} \quad , \tag{3.3}$$

where $G_H^{(10-d)}$ is the internal metric in the heterotic-string frame, and there is a crucial sign change at $d = 6$. The independence of ϕ_H and ϕ_I in $d = 6$ is not a surprise [13]. From the rational construction of [5], as well as from the orbifold approach in [6, 7], it is well known that type I models exist with different numbers of tensor multiplets, a setting with no analogue in perturbative heterotic compactifications on $K3$. Moreover, open descendants of type IIB Gepner models lead naturally to low-energy spectra without tensor multiplets [16]. In these cases the dilaton belongs to a hypermultiplet to be identified, on the heterotic side, with one of the moduli of the $K3$ compactification. In four dimensional $N = 1$ models, the dilaton lies in a linear multiplet on both sides, and heterotic - type I duality appears to be related to chiral - linear duality. The presence of an anomalous $U(1)$ suggests that R-R fields, that flow in the transverse channel and can thus take part in a generalized Green-Schwarz mechanism [8], correspond to charged scalars on the heterotic side.

4 A Candidate Heterotic dual

In order to substantiate these arguments, we now construct a heterotic model with (almost) the same perturbative spectrum as the open-string model of the previous section.

This heterotic model corresponds to compactification on the Z orbifold with non-standard embedding of the spin connection into the $SO(32)$ gauge group. The twist needed to achieve the desired result consists of four copies of the basic twist $(1/3, 1/3, 1/3)$ [4] and clearly satisfies the level matching constraints of modular invariance. As a result,

$SO(32)$ is broken to $SO(8) \times U(12)$, the CP group of the type I model. Moreover, the untwisted charged spectrum coincides with the open-string spectrum of the type I model. Indeed, the twist splits the heterotic R-moving fermions, $\tilde{\lambda}^I$, into three sets. Those inert under the twist, $\tilde{\lambda}_a$, with $a = 1, \ldots, 8$, those that pick a phase ω, $\tilde{\lambda}_r$, and those that pick a phase $\bar{\omega}$, $\tilde{\lambda}_{\bar{r}} = \delta_{\bar{r}s}\tilde{\lambda}^s$, with $r, s = 1, \ldots, 12$. The emission vertices of the charged massless scalars in the $(\mathbf{8}, \mathbf{12}^*)$ and $(\mathbf{1}, \mathbf{66})$ are $V^{ir}{}_a = \psi_i \tilde{\lambda}^r \tilde{\lambda}_a$ and $V^i_{[rs]} = \psi_i \tilde{\lambda}_r \tilde{\lambda}_s$, respectively.

A striking feature of the heterotic model is that twisted massless scalars are charged with respect to the gauge group. To show this, it proves very convenient to exploit the conformal embedding of $U(12)$ into $SO(24)$ at level one induced by the branching $\mathbf{24} = \mathbf{12_1} + \mathbf{12^*_{-1}}$. Denoting by X_n the characters of the integrable representations based on the n-fold antisymmetric products of the fundamental representation of $SU(12)$ and by ξ_m the $N = 2$ superconformal characters defined before eq. (2.6), one finds the decomposition

$$O_{32} + S_{32} = O_8 O_{24} + V_8 V_{24} + C_8 C_{24} + S_8 S_{24} = B_0 + B_+ + B_- \quad , \tag{4.1}$$

where

$$\begin{aligned} B_0 &= O_8(X_0\xi_0 + X_6\xi_6) + C_8(X_0\xi_6 + X_6\xi_0) + V_8(X_3\xi_{-3} + X_9\xi_3) + S_8(X_9\xi_{-3} + X_3\xi_3) \\ B_+ &= O_8(X_2\xi_{-2} + X_8\xi_4) + C_8(X_8\xi_{-2} + X_2\xi_4) + V_8(X_{11}\xi_1 + X_5\xi_{-5}) + S_8(X_{11}\xi_{-5} + X_5\xi_1) \\ B_- &= O_8(X_{10}\xi_2 + X_4\xi_{-4}) + C_8(X_4\xi_2 + X_{10}\xi_{-4}) + V_8(X_1\xi_{-1} + X_7\xi_5) + S_8(X_1\xi_5 + X_7\xi_{-1}) \ . \end{aligned} \tag{4.2}$$

Embedding the Z_3 twist in the Z_{12} center of $SU(12)$ assigns a phase ω^ϵ to $X_{3k+\epsilon}$, with $\epsilon = 0, \pm 1$. An S modular transformation on the resulting projection $B_0 + \omega B_+ + \bar{\omega} B_-$ yields the R-moving twisted sector. Combining it with the L-moving sector and taking into account the Casimir energy $\Delta E = 1/3$, one obtains massless particles only from $O_8 X_0 \xi_{-4}$ and $C_8 X_0 \xi_2$. The 27 blowing up modes of the fixed points, associated with $O_8 X_0$, that together with the 9 complex scalars from the untwisted sector represent the moduli of the Z orbifold, are thus charged ($m = -4$ in the chosen units) with respect to the anomalous $U(1)$. This is precisely what we are suggesting for the massless R-R scalars from the unoriented closed string spectrum in the type I model. The only difference between the

two low-energy spectra lies in the presence, in the heterotic case, of additional chiral multiplets in the $(\mathbf{8}_c, \mathbf{1})$ representation of the surviving gauge group, $SO(8) \times SU(12)$.

5 Low Energy Interactions

One can easily compute the β functions of the $SO(8)$ and $SU(12)$ factors, and find $\beta^{SO(8)} = 9$, $\beta^{SU(12)} = -9$. The $SO(8)$ factor, not asymptotically free, is not necessarily problematic in String Theory, since one has an effective ultraviolet cutoff, M_s, of the order of the Planck mass. The $SU(12)$ interactions may be responsible for interesting non-perturbative effects [18]. Denoting by Φ_b^{is} and $\chi_{[rs]}^k$ the chiral multiplets in the $(\mathbf{8}, \mathbf{12}^*)$ and $(\mathbf{1}, \mathbf{66})$ representations of the CP group, the cubic superpotential for the charged matter fields is fixed by gauge symmetry to be

$$W = y \; \delta^{ab} \; \epsilon_{ijk} \; \Phi_a^{ir} \Phi_b^{js} \chi_{[rs]}^k \quad . \tag{5.1}$$

The Yukawa coupling y is a constant, independent of the moduli fields of the closed-string spectrum, since the matter fields belong to the untwisted sector of the Z orbifold [19]. The manifest global $SU(3)$ symmetry is expected to be broken by higher-order terms, that we neglect for the time being. The other renormalizable interactions are encoded in the D-terms. Denoting by A^i and F^i the v.e.v.'s of the scalar fields in the chiral multiplets χ and Φ and assuming minimal kinetic terms, one gets

$$D = \sum_i 2\mathrm{tr}(A_i^\dagger A^i) - \mathrm{tr}(F^i F_i^\dagger) \tag{5.2}$$

for the anomalous abelian factor,

$$D^A = \mathrm{tr}(F^i T^A F_i^\dagger) = (\mathbf{T}^A)^a{}_b D^b{}_a \tag{5.3}$$

for $SO(8)$, where $A = 1, \ldots, 28$ and $\mathbf{T}^A$ are generators in the vector representation, and

$$D^I = \sum_i 2\mathrm{tr}(A^i \mathbf{t}^I A_i^\dagger) - \mathrm{tr}(F_i^\dagger \mathbf{t}^I F^i) = (\mathbf{t}^I)_r{}^s D_s{}^r \tag{5.4}$$

for $SU(12)$, with $I = 1, \ldots, 143$ and $\mathbf{t}^I$ the generators in the anti-fundamental representation.

In order to investigate further breakings of the gauge symmetry associated with flat directions of the scalar potential, the D-term conditions $D^A = 0$ and $D^I = 0$ must be supplemented with the F-term conditions

$$\begin{aligned}\frac{\partial W}{\partial F^{ir}_a} &= 2y\epsilon_{ijk}F^{js}_a A^k_{[rs]} = 0\\ \frac{\partial W}{\partial A^k_{[rs]}} &= y\epsilon_{ijk}\delta^{ab}F^{ir}_a F^{js}_b = 0 \quad .\end{aligned} \tag{5.5}$$

The condition $D = 0$ is not to be imposed, because of the one-loop Fayet-Iliopoulos D-term. For one family, similar problems have been studied in the literature [18]. In our model with three families, with no loss of generality one can set

$$F^1 = \begin{pmatrix} f^1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad A^1 = \begin{pmatrix} a^1 & 0 \\ 0 & b^1 \end{pmatrix} \quad , \tag{5.6}$$

where f^1, $a^1 = -(a^1)^T$ and $b^1 = -(b^1)^T$ are (generically invertible) complex matrices of dimension 8×8, 8×8 and 4×4 respectively. The F-term conditions then imply that all the F^i and A^i are of the same form as (5.6). f^i and a^i can then break $SO(8)\times SU(8)$ to $U(1)^4$, compatibly with the F and D term conditions above. On the other hand, the three b^i, that belong to the **6** (complex vector) representation of $SU(4) \sim SO(6)$, generically break it to $SU(2)$.

Non-perturbative effects are expected to generate corrections to the tree-level superpotential. Instanton calculus in the original $SU(12)$ is not adequate in this case since $N_f = 3 > \frac{N_c}{N_c - 3} = 4/3$, [18]. Holomorphy, symmetry arguments and limiting behaviors, however, are powerful tools to constrain the low-energy effective lagrangian [20]. In particular, the appearance of the $SU(12)$ gauge singlet chiral multiplets $U = \chi\Phi^2$, already present in the tree-level superpotential, as well as $V = \chi^{30}\Phi^{24}$, expected in the dynamically generated superpotential, seems unavoidable. Other composite chiral fields, as well as real superfields, may be needed to gauge part of the original flavor symmetry and to achieve a consistent low-energy description [20].

6 Comments

Let us discuss the effect of a quantized NS-NS antisymmetric tensor background, B_{ab}, in the orbifold planes. The quantization condition, related to the requirement of left-right symmetry in the toroidal compactification, was discussed in [9], where it was found that a B_{ab} of rank r reduces the CP multiplicity to $32/2^{\frac{r}{2}}$. Up to now, in order to keep the CP group as large as possible, we have discarded this possibility. However, it is simple to construct a rational model based on the Z_3 orbifold of the E_6 maximal torus [21], that requires a B_{ab} with $r = 6$. The type IIB parent theory is built out of 27 generalized characters, Y_k, with identity $Y_0 = A_0\chi_0^4 + A_+\chi_-^4 + A_-\chi_+^4$, that expose a global $SU(3)^4$ symmetry. The diagonal and the charge-conjugation modular invariant lead to different open descendants, but in both cases the maximal CP group is $SO(4)$. In the charge conjugation case, the massless closed spectrum includes 36 linear multiplets, while the open spectrum is pure super Yang-Mills. In the diagonal case, the massless closed spectrum includes 12 vector multiplets and 24 chiral multiplets, while the open spectrum comprises 27 chiral multiplets in the adjoint representation of the CP group, so that the resulting theory is weakly coupled at low-energies. Other choices of modular invariant torus amplitudes result in different numbers of vector, linear and chiral multiplets in the closed spectrum, as well as in different patterns of CP symmetry breaking. All these models are not chiral. The intermediate possibilities for B_{ab}, $r = 2$ and $r = 4$, correspond to non-vanishing tensor background in one or two orbifold planes, and lead to CP groups $SO(8) \times U(4)$ (with three generations in the $(\mathbf{8}, \mathbf{4})$ and $(\mathbf{1}, \mathbf{10})$) and $U(4)$ (with three generations in the $\mathbf{6}$).

Phenomenological prospects for type I vacua where not among the purposes of this work. It is conceivable, however, that some chiral type I vacuum may accommodate the standard model of particle physics. The natural appearance of three generations in the Z_3 case is rather amusing in this respect, and the symmetry breaking pattern along flat directions is very rich. Thus, one may embed a phenomenologically more appealing $SU(5)$ in the $SU(12)$ factor of the CP group. However, it seems rather difficult to get rid of the remaining $SO(8)$ and to obtain a minimal spectrum of $\mathbf{10}$'s and $\mathbf{5}^*$'s.

Acknowledgments

It is a pleasure to thank I. Antoniadis, C. Bachas, S. Ferrara, and G.C. Rossi for stimulating discussions. A.S. would like to acknowledge the kind hospitality of the CERN Theory Division while this work was being completed. This work was supported in part by E.E.C. grant CHRX-CT93-0340.

References

[1] A. Sagnotti, *in* Cargese '87, "Non-Perturbative Quantum Field Theory", eds. G. Mack et al (Pergamon Press, Oxford, 1988), p. 521; P. Horava, *Nucl. Phys.* **B327** (1989) 461, *Phys. Lett.* **B231** (1989) 251.

[2] D. Gepner, *Nucl. Phys.* **B296** (1988) 757; Lectures at the 1989 Trieste Spring School, **PUPT**-1121; T. Eguchi, H. Ooguri, A. Taormina and S.-K. Yang, *Nucl. Phys.* **B315** (1989) 193.

[3] I. Antoniadis, C. Bachas and C. Kounnas, *Nucl. Phys.* **B288** (1987) 87; I. Antoniadis and C. Bachas, *Nucl. Phys.* **B298** (1988) 586; K.S. Narain, M.H. Sarmadi and C. Vafa, *Nucl. Phys.* **B288** (1987) 551; H. Kawai, D.C. Lewellen and S.-H. Tye, *Nucl. Phys.* **B288** (1987) 1; W. Lerche, D. Lüst and A.N. Schellekens, *Nucl. Phys.* **B287** (1987) 477.

[4] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B261** (1985) 678; *Nucl. Phys.* **B274** (1986) 285.

[5] M. Bianchi and A. Sagnotti, *Phys. Lett.* **B247** (1990) 517; *Nucl. Phys.* **B361** (1991) 519.

[6] E. Gimon and J. Polchinski, hep-th/9601038.

[7] A. Dabholkar and J. Park, hep-th/9602030, hep-th/9604178; E. Gimon and C.V. Johnson, hep-th/9604129.

[8] A. Sagnotti, *Phys. Lett.* **B294** (1992) 196; M.J. Duff, R. Minasian and E. Witten, hep-th/9601036;

N. Seiberg and E. Witten, hep-th/9603003;
D.R. Morrison and C. Vafa, hep-th/9602114, hep-th/9603161;
S. Ferrara, R. Minasian and A. Sagnotti, hep-th/9604097.

[9] M. Bianchi, G. Pradisi and A. Sagnotti, *Nucl. Phys.* **B376** (1992) 365.

[10] G. Pradisi and A. Sagnotti, *Phys. Lett.* **B216** (1989) 59.

[11] S. Ferrara and S. Sabharwal, *Nucl. Phys.* **B332** (1989) 317.

[12] E. Witten, *Phys. Lett.* **B149** (1984) 351;
M. Dine, N. Seiberg and E. Witten, *Nucl. Phys.* **B289** (1987) 317.

[13] D. Berkooz, R. Leigh, J. Polckinski, J.H.Schwarz, N. Seiberg and E. Witten, hep-th/9605184.

[14] E. Witten, *Phys. Lett.* **B155** (1985) 151.

[15] A. Tseytlin, hep-th/9510173, hep-th/9512081.

[16] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, in preparation.

[17] E. Witten, *Nucl. Phys.* **B443** (1995) 85;
C. Vafa and E. Witten, hep-th/9507050.

[18] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, *Phys. Rep.* **162** (1988) 169.

[19] L. Dixon, D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys.* **B282** (1987) 13;
S. Hamidi and C. Vafa, *Nucl. Phys.* **B279** (1987) 465.

[20] K. Intriligator and N. Seiberg, "Lectures on Supersymmetric Gauge Theories and Electric-Magnetic Duality", hep-th/9609066.

[21] M. Bianchi, *Ph.D.* Thesis, **ROM2F**-92/13.